1. INTRODUCTION

A concrete shell, also commonly called thin shell concrete structure, is a structure composed of a relatively thin shell of concrete, usually with no interior columns or exterior buttresses. The shells are most commonly flat plates and domes, but may also take the form of ellipsoids or cylindrical sections, or some combination thereof.

All buildings are meant to enclose spaces. Most of the different types of superstructures we commonly use for present day building are only a modification of the age old system of column, beam and roof covering arrangements. They fulfill their function by two separate systems. One is the space covering system to cover the space, such as concrete slab or “roof covering sheets” in steel building. These are supported by a second system of beams and columns which we may call the supporting system. In many steel buildings they are obviously separate and in R.C buildings also, they are treated as two separate systems. In reinforced concrete shells, however, the two functions of covering the space and supporting the covering system are integrated into one. The structure covers the space without beams and columns within the buildings.

Structure as a shell form:

Thin shells are tan example of strength through from as opposed to strength through mass. The effort in the design is to make the shell as thin as practical requirements will permit so that the dead weight is n reduced and the structure functions as a membrane free from the large bending stresses. By this means, a minimum of materials is used to the maximum structural advantage. Shells of eggs, nuts and the human skull are commonplace examples. These naturally occurring shells are hard to crack or break.

1.1. Shells in Engineering:

Thin Shells as structural elements occupy a leadership position in engineering and especially in civil engineering, since they can be used in the construction of large span roofs, liquid retaining structures, domes folded plates and so on.
1.2. Advantages of using shell structures:

1) The efficiency in its load carrying behavior (being treated as a membrane)

2) High degree of reserved strength and structural integrity

3) High Strength: weight ratio (which is the main criteria in measuring structure efficiency)

4) Very High Stiffness

5) Containment of Space

6) High Aesthetic Value

1.3. Disadvantages:

It’s impossible to build a story above the story that has a shell roof, thus shells are always used as a “terminating” roof.

1.4. MAIN SHELL TYPES:

- Spherical Shells (Dome structures)

- Cylindrical Shell

- Hyperbolic Parabloid Shell

- Toroidal Shells

2. CYLINDRICAL SHELL:

Reinforced concrete cylindrical shell roofs are chosen commonly for covering large clear areas using the minimum of intermediate supports, such as in factory buildings, godowns, power station, garages, island platforms of railway stations, stadia etc.,
Types of cylindrical shell:

- Single Barrel Cylindrical shell
- Continuous cylindrical shells
- Butterfly shells
- Corrugated shells

SINGLE BARREL CYLINDRICAL SHELL:

Circular cylindrical shells are generally classified as long and short shell. The geometrical characteristics of the shell justify these shells are long or short. Long shells were very popular for roofs of factories and short shells for aircraft.

2.1. Forming of Cylindrical Shells:
Cylindrical Shell can be thought of a surface generated by a straight line (generator) moving over a plane curve (directrix)

The major force acting on the cylindrical shell is as shown in fig.

$T_x =$ Direct force (tension or compression) in the longitudinal direction. (kg/m.)

$T_\theta =$ Direct force in the $\theta$ (transverse) direction. (kg/m.)

$S =$ Tangential shear force considered positive when it provides tension in the direction of increasing value of $x$ and $\theta$. (kg.)
MØ = Bending moment in radial face is taken as positive when it produce tension inside the shell. (kgm.)

2.2. Selection shell:

**Length and chord width**: long cylindrical shells $L \geq 3B$ Length of the span is $(L)$ 30M and chord width is $(B)$ 10M.

Main Characteristics:

- Deformation of cross section is small
- Both Membrane Theory and Beam Analysis Applies

**Radius**: The radius of cylindrical shell has to be chosen keeping acoustic consideration in view. It desirable to see that the centre of curvature does not lie at the working level.

**Semi central angle**: The semi central angle should be between 30 and 45 degrees to facilitate easy concreting (without back form). If the angle exceeds 45°, concreting becomes difficult without the use of top form. If the angle is 40°, wind load can be ignored, because it causes only suction on the shell. Between the limits of 30° and 40°, it is desirable to choose as large central angle possible with the object of getting a high structural depth for the shell.

So that the semi central angle ($Ø$) is 40°.

**Thickness of the shell**: Thickness of the shells shall not normally be less than 50mm is single curved and 40mm if doubly curved. The usual thickness of the singly curved cylindrical shell is between 70mm and 80mm.

Selected thickness of the shell is 80mm.

**Structural depth** (central rise): For single shells the total depth between the crown of the shell and the top of the edge beam shall be between one fifth and one twelfth of the smaller span.
\[ x = \frac{5}{\tan 40^\circ} = 6 \text{m} \]
\[ R = \sqrt{5^2 + 6^2} = 7.8 \text{m} \]
\[ h = 7.8 - 6 = 1.8 \text{m} \]

2.3. Stresses in a Cylindrical Shell

Membrane Theory

Thin shells are assumed to work as a membrane for its low bending rigidity, thus no bending moments develop and the force is resisted entirely by both membrane and ring forces acting on the shell.

For a shell to be classified as “thin” it must satisfy the following criteria:

\[ \text{Max } (h/R) \leq 1/20, \]

Where: 
- \( h \) = shell thickness
- \( R \) = shell radius of curvature

Membrane vs. Beam Theory


2.4. **Edge beam:**

The design of edge beam is a very important component in cylindrical shells. The longitudinal tension at the edges will be very large. It is advisable to provide an edge beam to accommodate this large tension by a vertical shallow edge beam. This beam is usually only about 1/12 to 1/16 of span in depth so that it acts as a shallow beam

\[
\text{Depth of the edge beam} = \frac{\text{span}}{12} \text{ to } \frac{\text{span}}{16}
\]

\[
= \frac{30}{12} \text{ to } \frac{30}{16}
\]

\[
= 2.5\text{M to } 1.875\text{M} \approx 1.8\text{M}
\]

Width of edge beam = Three times of the shell thickness.

\[
= 3 \times 0.080 = 0.240 \approx 0.300\text{M}
\]

\[
\therefore \text{The depth of the edge beam} = 1.8\text{M}
\]

\[
\text{Width} = 0.300\text{M}
\]
2.5. Diaphragm:

The Diaphragm is considered the support of a cylindrical shell and is subjected to its shear and gravity loads. To analyze it, we need to consider two loads:

1) The direct gravity loads: consists of the loads transferred from the shell to the diaphragm as well as the diaphragm’s self weight.

2) The indirect Loads that are transferred due to shear stresses. These loads are transferred in the form of Horizontal Components and Vertical Components. And are obtained by dividing the diaphragm into small elements as shown in the next figure.

\[ F = \text{Shear. } ds \]
\[ V = F \sin \varnothing \]
\[ H = F \cos \varnothing \]

The Moment is calculated by adding the normal static moment and the moment induced by the additional V and H forces.

Once all forces are obtained, the diaphragm is treated as a beam (or beam column).
**Function of a transverse:**

The supports provided along the curve edge of a shell are known as transverse. They are meant to preserve the assumed geometry of the shell. A developable surface, such as a cylinder, will flatten out under loads in the absence of the transverse. Hence it is indispensable part of a cylindrical shell.

**Types of Transverse:**

- Solid diaphragm
- Arch rib
- Tied arch
- Bow string girder
- Portal frame

### 2.6. Dimensions and recommendations:

- **Span length (l)** = 30 m
- **Chord width (B)** = 10 m
- **Radius (R)** = 7.80 m
- **Semicircular angle (Ø)** = 40°
- **Thickness of the shell (t)** = 80mm
- **Central rise (h)** = 180cm
- **Arc length** = 10.90m

**Types of cylindrical shell:**

\[
\rho = \left( \frac{12\pi R^6}{L^4 d^2} \right)^{1/8}
\]

\[
= \left( \frac{12 \times \pi^4 \times 7.8^6}{30^4 \times 0.078^2} \right)^{1/8}
\]

\[
= 3.899 < \bar{X}
\]
And, \( K = \frac{\pi^2 R^2}{L^2 p^2} \)

\[ = \frac{\pi^2 \times 7.6^2}{30^2 \times 3.89^2} = 0.044 < 0.12 \]

\[ = 0.044 < 0.12 \]

Span length is three times of the chord width (\( L \geq 3B \))

Hence long shell (from IS 2210-1988)

3. **ANALYSIS OF CYLINDRICAL SHELL:**

3.1. **Load calculation:**

Dead load (0.08 \( \times \) 24) \( = 1.92 \text{ KN/m}^2 \)

Weather proofing \( = 0.23 \text{ KN/m}^2 \)

Total dead load \( = 2.15 \text{ KN/m}^2 \)

Live load \( (P_L) = 0.75 \text{ KN/m}^2 \) (from the code book recommends)

Dead load is taken as uniform along the circumference of the shell and live load as uniform along the horizontal projection of the shell. Dead load which acts on the surface of the shell, the live load prescribed acts on the plan area of the shell as shown in figure.

\[ \text{Dead load} \quad + \quad \text{Live load} \quad = \quad \text{Equ. load} \]
Equivalent Dead load

\[ P_d = P_1 \left( \frac{\sin \phi k}{\phi k} \right) \]

\[ = P_L \left( \frac{\sin 40^\circ}{0.698} \right) = 0.70 \text{ KN/m}^2 \]

Total load = Dead load + Live load

\[ = 2.15 + 0.70 \]

\[ = 2.85 \text{ KN/m}^2 = 291 \text{ kg/m}^2 \]

For sign loading increment factor \( = \frac{4}{\pi} \times p' \)

\[ = \frac{4}{\pi} \times 291 \text{ kg/m}^2 \]

\[ = 370 \text{ kg/m}^2 \]
PART I

3.2. MEMBRANE ANALYSIS:

Membrane force $T_x$ at $x = \frac{L}{2}$

$$T_x = P_u r \left( \frac{L}{r} \right)^2 \left[ \cos(\phi_k - \phi) \times \frac{2}{n^2} \right]$$

Constant $= P_u r \left( \frac{L}{r} \right)^2 = 370 \times 7.8 \times \left( \frac{3\phi}{7.8} \right)^2$

$= 33577$

Coefficient $= 0$ {From ASCE manual Table 1b}

Hence, $T_x = \text{constant} \times \text{coefficient}$

This value is tabulated in Table 1

Membrane force $S$ at $x = 0$

$$S = P_u r \frac{L}{r} \left[ \sin(\phi_k - \phi) \times \frac{2}{n^2} \right]$$

Constant $= P_u r \times \frac{L}{r} = P_u \times L = 370 \times 30 = 8730$

Coefficient $= \phi = 0$ (or) $(\phi_k - \phi) = 40^\circ$

$S = \text{constant} \times \text{Coefficient}$

This value is given in Table 2

Membrane force $T\phi$ at $x = \frac{L}{2}$

$$T\phi = P_u r \cos(\phi_k - \phi)$$

Constant $= P_u r = 370 \times 7.8 = 2269$

Coefficient $= (\phi_k - \phi) = 0$, then value $= -1$

$T\phi = \text{Constant} \times \text{Coefficient}$

This value is tabulated in Table
VALUE OF Mø:

There is no bending in membrane analysis but this will be produced when we apply correction edge forces.

These calculation are made for (φ_k - φ) value 0,10,20,30&40

This value is tabulated in Table 4

PART II

3.3. CORRECTION ANALYSIS:

Correction forces = V_L, H_L and S

\[ T\varphi = (P_u \times r) \times \cos 40^\circ \]

\[ = 2269 \times \cos 40^\circ \]

\[ = 1738 \text{ kg/m.} \]

Then line loads (correction forces):

\[ V_L = T\varphi \sin \varphi_k = 1738 \times \sin 40^\circ \]

\[ H_L = T\varphi \cos \varphi_k = 1738 \times \cos 40^\circ \]

\[ S = \text{constant} \times \text{co efficient} = 8730 \times 0.4092 = 3572 \]

For T_x caused by application of V_L, H_L, S

\[ T_x \text{ due to corrective load } V_L = V_L \times \left(\frac{L}{r}\right)^2 \times \text{co efficient.} \]

\[ = 1117 \times 14.8 \times \text{co eff} \]

\[ T_x \text{ due to corrective load } H_L = 1331 \times 14.8 \times \text{co eff} \]

\[ T_x \text{ due to corrective load } S = 3572 \times 14.8 \times \text{co eff} \]
For $S$ to be caused by application of $V_L, H_L, S$

S due to corrective load $V_L = V_L \times \left(\frac{L}{L'}\right) \times \text{coeff} = 1117 \times 3.8 \times \text{coeff}$

S due to corrective load $H_L = 1331 \times 3.8 \times \text{coeff}$

S due to corrective load $S = 3572 \times 3.8 \times \text{coeff}$

For $M_\theta$, there is no moment in membrane analysis due to $V_L, H_L,& S$:

$M_\theta$ due to corrective load $V_L = 1117 \times 7.8 \times \text{coeff}$

$M_\theta$ due to corrective load $H_L = 1331 \times 7.8 \times \text{coeff}$

$M_\theta$ due to corrective load $S = 3572 \times 7.8 \times \text{coeff}$

**TABLE 1:**

<table>
<thead>
<tr>
<th>$T_x$</th>
<th>$(\phi_k - \phi)$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

| Membrane load | $33577 \times \text{co eff}$ | -6803 | -6702 | -6393 | -5889 | -5211 |

| LINE LOADS | $V_L=1117 \times 14.8 \times \text{coeff}$ | -3571 | -16862 | -36534 | -4876 | +165481 |
| $H_L=1331 \times 14.8 \times \text{coeff}$ | -23165 | -9583 | +18006 | +21136 | -60278 |
| $S=3572 \times 14.8 \times \text{coeff}$ | -8818 | -6565 | +1390 | +17985 | 47320 |
| | | | | | | |
| | | -42357 | -39692 | -23531 | 28329 | 147222 |
### TABLE 2:

<table>
<thead>
<tr>
<th>s</th>
<th>$(\Phi_k - \Phi)$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Membrane load

| 8730×coeff | 0  | -964 | -1901 | -2778 | -3572 |

LINE LOADS

| $V_L=1117\times3.8\timescoeff$ | 0  | -1167 | -5106 | -9087 | 0   |
| $H_L=1331\times3.8\timescoeff$ | 0  | -2604 | -2001 | 1353  | 0   |
| S=3572×3.8×coeff | 0  | -1136 | -1579 | -342  | 4072 |

| 0  | -5871 | -10587 | -10854 | -500  |

### Table 3:

<table>
<thead>
<tr>
<th>TØ</th>
<th>$(\Phi_k - \Phi)$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Membrane load

| 2269×coeff | -2269 | -2234 | -2132 | -1964 | -1738 |

LINE LOADS

| $V_L=1117\times7.8\timescoeff$ | -2208 | -2113 | -1597 | -377  | +719  |
| $H_L=1331\times7.8\timescoeff$ | 896   | 1092  | 1422  | 1384  | 1019  |
| S=3572×7.8×coeff | -262  | -173  | 4465  | 218   | 0     |

| -3843 | -3428 | -2262 | -739  | 0     |
Table 4:

<table>
<thead>
<tr>
<th>Mφ</th>
<th>(φk - φ)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td></td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Membrane load
2269×coeff

| LINE LOADS | V_L=1117×7.8×coeff | -2485 | -2292 | -1726 | -866 | 0  |
| H_L=1331×7.8×coeff | 1612 | 1544 | 1299 | 791 | 0  |
| S=3572×7.8×coeff | -133 | -108 | -50 | -2 | 0  |
|              | -1006 | -856 | -477 | -97 | 0  |

THE FINAL VALUES OF T_x, S, T_ø, M_ø

<table>
<thead>
<tr>
<th>φ</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>(φk - φ)</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>T_x</td>
<td>147222</td>
<td>28239</td>
<td>-23531</td>
<td>39692</td>
<td>-42357</td>
</tr>
<tr>
<td>S</td>
<td>-500</td>
<td>-10854</td>
<td>-10587</td>
<td>-5871</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>0</td>
<td>-739</td>
<td>-2262</td>
<td>-3428</td>
<td>-3843</td>
</tr>
<tr>
<td>M_ø</td>
<td>0</td>
<td>-97</td>
<td>-477</td>
<td>-856</td>
<td>-1006</td>
</tr>
</tbody>
</table>
CHECK 1:

Find $\Delta L$

$$\frac{\Delta L}{3} = \frac{2\pi r \times 10}{360} = \frac{2 \times 3.14 \times 7.8}{360} \times 10$$

$$= 1.36 \approx 1$$

$$\sum T_x = \frac{2 \times \Delta L}{3} [\text{First} + 4 \sum y_1 + 2 \sum y_2 + \text{Last}]$$

$$= \frac{2 \times 1.36}{3} [147222 + (4 \times 128329) + (2 \times -23531 +)

(4 \times -39692) + (-42357)]$$

$$= 0.906 \times [260538 - 248187]$$

$$= 0.906 \times 12351$$

$$= 11190 \text{ kg/m.}$$

Hence, Difference $= \frac{11190}{260538} = 0.0429$

$$= 4.1\% \text{ only.}\$$

HENCE OK.

CHECK 2:

FIND THE SUM OF VERTICAL COMPONENTS:

$$S = \frac{\Delta L}{3} [Y_0 + 4Y_1 + 2Y_2 + \ldots + Y_n]$$

$$= \frac{1.36}{3} [-500 + 4 \times -10854 + 2 \times -10587 +

4 \times -5871 + 0]$$

$$S = 0.453[-500-43416-21174-23484]$$

$$S = 40124 \text{ Kg (name as A)}$$
Find the total shear at support:

\[ W_{\text{dead}} = \frac{370 \times (2 \pi \times 7.8) \times 30}{360} = 3169.20 \]

End reaction \( = \frac{4L}{\pi^2} \times 3169.2 = \frac{4 \times 30}{\pi^2} \times 3169.2 \)

\( = 38532 \text{ Kg (name as B)} \)

\( A \approx B \)

i.e, 4.1% only

**CHECK 3:**

Calculate the lever arm from bottom of the shell

i) \( (\phi_k - \phi) = 0, \phi_k = 40 \)

Lever arm = \( r[\cos(\phi_k - \phi) - \cos\phi_k] \)

\( = 7.8[1 - \cos 40] = 1.80 \text{M} \)

ii) \( (\phi_k - \phi) = 10, \phi_k = 30 \)

\( = 7.8[\cos 10 - \cos 40] \)

\( = 1.71 \text{M} \)

iii) \( (\phi_k - \phi) = 20 \)

\( = 7.8[\cos 20 - \cos 40] \)

\( = 1.35 \text{M} \)

iv) \( (\phi_k - \phi) = 30 \)

\( = 7.8[\cos 30 - \cos 40] \)

\( = 0.77 \text{M} \)

v) \( (\phi_k - \phi) = 40 \)

\( = 7.8[\cos 40 - \cos 40] = 0. \)
VALUES OF FORCES AT 10° INTERVAL:

<table>
<thead>
<tr>
<th>$\phi_k - \phi$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_x$</td>
<td>-42357</td>
<td>-39629</td>
<td>-23531</td>
<td>28329</td>
<td>147222</td>
</tr>
<tr>
<td>LEVER ARM</td>
<td>1.80</td>
<td>1.71</td>
<td>1.35</td>
<td>0.77</td>
<td>0</td>
</tr>
</tbody>
</table>

INTERNAL MOMENT:

$$M_1 = \frac{2\Delta L}{3} [0 + (4 \times 0.77 \times 28329) + (2 \times 1.35 \times 23531) + (4 \times 1.71 \times -39692) + (1.82 \times 42357)]$$

$$M_1 = 294542.816 \text{ kgm.}$$

EXTERNAL MOMENT:

Weight per meter length = 3169 Kg/m

$$M_2 = \frac{4}{\pi^2} (wL^2)$$

$$M_2 = \frac{4}{\pi^2} (3169 \times 30^2)$$

$$= 367938 \text{ kgm.}$$

Error between $M_1$ & $M_2$

$$= \frac{73336}{294542} = 0.24$$

Hence safe.
3.4. ANALYSIS EDGE BEAM:

**PART A**

**MEMBRANE ANALYSIS:**

Dead load = 2.15KN/m² = Fourier load = \(\frac{4}{\pi} \times 2.15 = 2.73\)KN/m²

Live load = 0.75KN/m² = Fourier load = 0.95 KN/m²

Where, Total load = 3.68 KN/m² \(\approx 380\) kg/m²

Combined dead load = 380 kg/m².

\(\frac{r}{t} = 100\) & \(\frac{r}{L} = 0.3\)

Find total \(T_x\) for membrane analysis:

\[
T_x = \left(\frac{L}{r}\right)^2 \times p \times r \times co.\ eff
\]

\[
T_x = \left(\frac{L}{r}\right)^2 \times 380 \times 7.8 \times -0.1552
\]

= \(\left(\frac{L}{r}\right) (-460)\)

Calculate \(T\) by membrane analysis and find \(V_L\) & \(H_L\):

\[
T = r (p \times co.\ eff) = 7.8 \times 370 \times (-0.7660)
\]

= -2210kg

\(V_L = -2210 \times \sin 40^\circ = -1420kg\)

\(H_L = -2210 \times \cos 40^\circ = -1693kg\)

Calculate \(S\) by membrane analysis:

\[
S = p \times r \left(\frac{L}{r}\right) \times co\ eff
\]

= 377 \times 30 \times -0.4092 = -4628 kg.
Write down the expression for total $T_x$ due to all for components,

$$T_x = \left(\frac{L}{r}\right)^2 [-460 - (1420 \times 10.01) - (1693 \times -3.608) + (4628 \times 0.8934)]$$

$$T_x = \left(\frac{L}{r}\right)^2 (12700)$$

These values of $T_x$ due to membrane analysis and $V_b$ & $S_b$ from beam effect (Add for $V_b$ & $S_b$)

$$T_x = \left(\frac{L}{r}\right)^2 [12700+10.01V_b+0.8934S_b].$$

PART II

3.5. CALCULATION OF DEFLECTION OF SHELL:

1. SUMMARY PROCEDURE:

We $\Delta V$ of the membrane analysis (Table 1B of manual no.31 and deflection due to $T_x$, $V_L$, $H_L$ and $V_b$, $S_b$ using Table II B of manual as follows)

$$\Delta V = \text{(membrane deflection)} + \text{(That due to application of } V_L, H_L \text{ and } S_b)$$

$$\Delta V = \frac{L^4}{r^3 t_E} \left[ \left(\frac{2r}{nL}\right)^2 + \frac{2}{n^2} + \left(\frac{r}{L}\right)^4 \times 0.6710 \right]$$

$$+ \frac{L^4}{r^3 t_E} [V_L(37.31) + H_L(-15.65) + S(0.9583)]$$

(Where $V_L = -1420$; $H_L = -1693$ and $S = 4628$

$$= \frac{L^4}{r^3 t_E} (25744)$$

Adding $V_b$ and $S_b$

$$\Delta V = \text{Total deflection} = \frac{L^4}{r^3 t_E} (25744 + 42.38 V_b + 0.9583 S_b)$$
To find $V_b$ & $S_b$ value:

\[
\frac{T_x}{t} = \left(\frac{r}{r}ight)^2 \times \left(\frac{1}{l}\right) \left[12700 + 10.01V_b + 0.8934S_b\right]
\]

2) Equating $\frac{T_x}{t}$ to an equation to stress at top of beam (flop) and reducing it to an equ.

Compatibility of stress

Shell + beam

\[
\text{Beam} = \left(\frac{r}{h}\right)^2 \times \left(\frac{1}{l}\right) \left[0.60793 \left(\frac{4}{\pi} \times w - V_b\right) + 1.273 \left(\frac{h}{l}S_b\right)\right]
\]

Beam weight = 0.3 × 1.2 × 2400 = 864 kg/m

Sinusoidal weight = $\frac{4}{\pi} \times w = 1100$ kg/m

\[
A + \left(\frac{r}{h}\right)^2 \times \left(\frac{1}{l}\right) \left[0.60793(1100 - V_b) + 1.273\left(\frac{h}{l}S_b\right)\right] = 0
\]

A + 3482 -3.166 + 0.3978$S_b$ = 0
\[ 12700 + 10.01V_b + 0.8934S_b + 3482 - 3.166 + 0.3978S_b = 0 \]

\[ A + \left( \frac{r}{h} \right)^2 \times \left( \frac{r}{b} \right) \left[ 0.60793 \left( 1100 - V_b \right) + 1.273 \left( \frac{h}{L} S_b \right) \right] = 0 \]

\[ 6.84V_b + 1.29S_b = -16182 \text{ (name as 'a')} \]

**Calculate vertical deflection:**

1) Due to membrane forces, Table 1A, 1B
2) Due to \( V_L, V_H \) and \( S \), Table 2B
3) Due to \( V_b \) and \( S_b \) on shell

We apply \(-V_b\) and \(-S_b\) on the beam.

Equating deflection on shell and beam:

\[ B + \left[ -\frac{t}{L} \left( \frac{r}{h} \right)^3 \left[ 0.12319 \left( 1100 - V_b \right) + 0.1935 \left( \frac{h}{L} S_b \right) \right] \right] = 0 \]

\[ B - 2940 + 2.67V_b - 0.252S_b = 0 \]

\[ 45.05V_b + 0.7041S_b = -22714 \text{ (name as 'b')} \]

Solve for \( a \) & \( b \):

\[ V_b = -336.6 \text{ kg} \]

\[ S_b = -10579 \text{ kg} \]

Calculate stress at top and bottom of beam due to total weight:

**Total vertical load = wt of beam + \( V_b \)**

\[ = 1100 + 336 \]

\[ = 1436 \text{ kg/m} \]

\[ f = \pm \left( 0.60793 \right) \left( \frac{L}{h} \right)^2 \times \frac{1}{b} \sin \frac{nx}{l} \]
Calculate stress in beam due to $S_b$:

Stress at top edge = $\frac{L}{bh} (1.2732) S_b$ (tensile)

$$= \frac{30}{0.3 \times 1.8} (1.2732) \times 10759 \text{ kg}$$

$$= 761019 \approx 76 \text{ kg/cm}^2$$

Stress as the bottom edge = 38 kg/m²

Resultant stress in beam due to $V_b + S_b$:

Compression at top beam = $80.8 - 76 = 4.8 \text{ kg/cm}^2$

Tension at top beam = $80.8 - 38 = 42.8 \text{ kg/cm}^2$

4. REINFORCEMENT DETAILS:

4.1. DESIGN OF STEEL REINFORCEMENT IN EDGE BEAM:

Depth of beam in tension: with above stress distribution 1.8 m

$$= \frac{180 \times 42.8}{47.6} = 161.84 \text{ cm}$$

Total tension = $\frac{1}{2} \times 161.84 \times 42.8 \times 30$

$$= 103875.6 \text{ kg}$$

Assuming 10 cm – cover

and stress at extreme fiber = 4500 kg/cm²
Allowable stress = \( \frac{4500 \times (161.84 - 10)}{161.84} \)

\[ = 4221.9 \text{ kg/cm}^2 \]

\[ \text{As} = \frac{103875}{4221.9} \]

\[ = 24.61 \text{ cm}^2 \]

This steel is provided in the tension zone.

Use 16 mm # bar,

\[ \frac{\text{ast}}{\text{Ast}} = \frac{2461}{\pi (10)^2} = 12.24 \]

\[ \approx 16 \text{ nos.} \]

Provide 16 mm # bar in 16 Nos.

Design of shear in steel:

Shear self weight = \( \frac{L}{2} \times w = \frac{30}{2} \times 864 = 12960 \text{ kg} \)

Shear in shell = 10854 kg

Total shear in beam (\( V_1 \)) = W + V

\[ = 12960 + 10854 \]

\[ = 23814 \]

\[ \therefore \text{fy} = 0.44 \times 250 \]

Spacing (s) = \( \frac{2 \times \text{As} \times 0.87 \times \text{fy}}{V_1} \)

\[ = \frac{2 \times 24.61 \times 0.87 \times 1120}{23814} \]

\[ = 29.61 \text{ cm}^2 \]

Provided two legged 16 mm # stirrups @ spacing 290 mm c/c.
4.2 DESIGN OF STEEL REINFORCEMENT IN SHELL:

Part I – Design for Tx force in the shell

Step: 1

Total rise of the shell = 180 cm

Neutral axis:

\[
N.A = \text{Rise} - (\text{radius} - y)
\]

\[
= 1.80 - (7.8 - 6.70)
\]

\[
= 70 \text{ cm}
\]

N.A = 70 cm from the Bottom.

Divide the depth below N.A into the following regions.

(a) Regions 1: From bottom to 20 cm above bottom

Allowable stress = 0.87 \( f_y \)

Use Fe 500

\[
= 0.87 \times 500 = 435 \text{ N/mm}^2
\]

\[
= \frac{435}{9.81} \times 435
\]

Allowable stress = 4500 kg/cm²

(b) Regions 2: From bottom 20 cm to 40 cm above bottom

\[
\text{Allowable stress} = \frac{4500 \times (70 - 20)}{70}
\]

\[
= 3214 \text{ kg/cm}^2
\]
(c) Regions 3: From 40 cm above bottom to NA

Allowable stress = \( \frac{4500 \times (70 - 40)}{70} \)

= 1928 kg/cm²

Step: 2

Steel area per meter length = \( \frac{\text{Tx force in region}}{\text{Allowable stress}} \)

(a) Region 1:

\[ Ast = \frac{147222}{4500} \]

Ast = 32.17 cm²

Use 16 mm # bar

\[ \frac{\pi (16)^2}{4} \times 1000 = 62.49 \approx 62.00 \text{ mm} \]

Provide 16 mm # bar @ 62 mm spacing c/c

(b) Region 2:

<table>
<thead>
<tr>
<th>0°</th>
<th>2°</th>
<th>10°</th>
</tr>
</thead>
<tbody>
<tr>
<td>147222</td>
<td>?</td>
<td>28329</td>
</tr>
</tbody>
</table>

\[ = 147222 + \left( \frac{28329 - 147222}{10 - 0} \right) \times 2 \]

= 123443.4 kg/m @ 2°

\[ Ast = \frac{123443.4}{4500} \]

Ast = 38.41 cm²
Use 16 mm # bar
\[ = \frac{2\pi d}{\pi d^2/4} \times 1000 \]
\[ = \frac{2 \times 16}{\pi (16)^2/4} \times 1000 \]
\[ = 52.49 \approx 52.00 \text{ mm} \]

Provide 16 mm # bar @ 52 mm spacing c/c

(c) Region 3:

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>5°</th>
<th>10°</th>
</tr>
</thead>
<tbody>
<tr>
<td>147222</td>
<td>?</td>
<td>28329</td>
<td></td>
</tr>
</tbody>
</table>

\[ = 147222 + \frac{(28329 - 147222)}{(10 - 0)} \]
\[ = 87775.44 \text{ kg/m @ 5°} \]

Ast = \[ \frac{87775.5}{4500} \]
Ast = 45.52 cm²

Use 16 mm # bar
\[ = \frac{2\pi d}{\pi d^2/4} \times 1000 \]
\[ = \frac{2 \times 16}{\pi (16)^2/4} \times 1000 \]
\[ = 50.19 \approx 50.00 \text{ mm} \]

Provide 16 mm # bar @ 50 mm spacing c/c

Nominal steel / meter length:

As (nomi) = \[ \frac{0.15 \times 100 \times 3.0}{100} = 1.2 \text{ cm}^2/\text{m} \]
\[ = 120 \text{ mm}^2/\text{m} \]
\[ = \frac{\pi \theta^2/4}{120} \times 1000 \]
\[ = 418.56 \approx 400 \text{ mm} \]
Spacing:

(i) \(4000 \text{ mm}\)
(ii) \(5 \times t = 5 \times 80 = 400 \text{ mm}\)
(iii) \(S = 450 \text{ mm}\)

Least value of above is \(S = 400 \text{ mm}\)
Provide 6 mm # bar @ 400 mm spacing c/c

**Part II – Design for TØ and MØ**

We will provide steel for TØ and MØ separately.

TØ is mostly compression. Use 10 mm rods.

Max. Force at \(Ø = 40^\circ\) is 2946 kg/m.

\[
\text{Stress} = \frac{3943}{100 \times 8.0} = 4.804 \text{ kg/cm}^2
\]

Design for Max. MØ = 1006 \times 100 \text{ cm.kg/m}

Effective depth = 8.0 - Clear cover - \(d/2\)

\[
= 8.0 - 2 - 0.5 = 5.5 \text{ cm}
\]

Lever arm = 0.87 \times 55 = 4.78 cm

\[
\text{Ast} = \frac{100600}{4500 \times (0.87 \times 4.78)} = 5.38 \text{ cm}^2/\text{m}
\]

Use 10 mm # bar

\[
\frac{a_{st}}{\text{Ast}} \times 1000 = \frac{\pi (12)^2 / 4}{\frac{538}{4}} \times 1000 = 212.98 \text{ mm} \approx 210 \text{ mm}
\]

Provide 12 mm # bar @ 210 mm spacing c/c.
Part III – Design for Shear (s)

Theoretically, the diagonals should be designed for principal tension. As an approximation,

let shear = tension

As the shear is placed at 45° (from the tension side as the diagonal steel for shear in a beam)

Divide shear at support into three regions,

(a) Steel for first region – Ø = 10° (at support)

We will assume, shear = tension

Shear @ 10° = 10854

\[ Ast = \frac{10854}{4500} = 2.4 \text{ cm}^2/\text{m} \]

Use 10 mm # bar,

\[ = \frac{\text{ast}}{\text{Ast}} \times 1000 \]

\[ = \frac{\pi(10)^2/4}{240} \times 1000 = 327.34 \]

\[ \approx 320 \text{ mm} \]

Provide 10 mm # bar @ 320 mm spacing c/c.

(b) Steel @ Ø = 20°

Shear @ 20° = 10587

Steel for tension = \( \frac{10587}{100 \times 8.0} \) = 13.23 kg/cm²

As the tension is low. We can adopt nominal steel.
5. DESIGN OF TRANSVERSE STIFFENERS OF CYLINDRICAL SHELL

Step: 1

Max. Depth of diaphragm = 180 + 180

= 360 cm

Width = 30 cm

Effective width of L-beam \((b_e) = 0.38\sqrt{rt}\)

\[= 0.38\sqrt{7.8 \times 0.08}\]

\[= 0.38\sqrt{7.8 \times 0.08} = 30 \text{ cm}\]

C.G of the section at centre

\[= \left(8 \times 30 \left(\frac{8}{2}\right) \right) + \left(360 \times 30 \times \left(\frac{360}{2}\right) \right) \]

\[\frac{(8 \times 30) + (360 \times 30)}{(8 \times 30) + (360 \times 30)}\]

\[= 176.1 \text{ cm} = 1.761 \text{ m from the top}\]

Step: 2

Total tension on the diaphragm due to shear:

TØ at 40° = 3843

TØ at crown of the shell = 3843 \(\sin \left(\frac{\pi x}{l}\right)\)

\[T = \int_0^{l/2} 3843 \sin \left(\frac{\pi x}{l}\right) \, dx = 36697.95 \text{ kg}\]

Moment due to \(T\) about

\[= 36697.95 \times 1.761\]

\[= 64635.09 \text{ kg/m.}\]
Step: 3  
Dead load and live from the shell to transverse:  
Circumference of the shell = RØ = 7.8 × 1.396  
= 10.90 m  
Dead load per m$^2$ of the shell = 0.08 × 2400  
= 192 kg/m$^2$  
Assume equivalent load due to  
live load along shell = 100 kg/m$^2$  
Total load due to 30m length  
of the shell = (192+100) × 10.90 × 30  
= 95000 kg  
(The moment due to this load produce tension at the base of the transverse.)  
Step: 4  
Shear and Bending moment due to L.L + D.L from shell resting on transverse.  
Shear = \( \frac{w}{2} = \frac{95000}{2} = 47500 \) kg
Bending moment = \( \frac{wl}{8} = \frac{9500 \times 10}{8} = 118750 \text{ kg} \)

Step: 5

B.M and Shear due to self weight of transverse:

Area of the transverse = Rect. Part + Segmental circle part

Rect. Part = \( 1.80 \times 0.3 \times 10 = 5.4 \text{ m}^3 \)

Segmental circle part = \( \frac{R^2}{2} (\theta - \sin\theta) \times 0.3 \)

\[ = \frac{7.8^2}{2} (1.396 - \sin 80^\circ) \times 0.3 = 3.75 \text{ m}^3 \]

Total volume = 5.4 + 3.75

\[ = 9.15 \text{ m}^3 \]

Total weight (w) = 9.15 \times 2400 = 21960 \text{ kg}

Bending Moment = \( \frac{21960 \times 10}{8} = 27450 \text{ kgm.} \)

Shear = \( \frac{21960}{2} = 10980 \text{ kg} \)

Step: 6

Design of shear:

Shear = 47500 + 10980 = 58480 \text{ kg}

Section at end (transverse) = 0.3 \times 1.8

Max. Shear stress = \( \frac{58480}{30 \times 180} \)

\[ = 10.83 \text{ kg/cm}^2 \]

[Max. Allowable for M20 concrete is about 18 kg/cm^2]
Design for shear as maximum allowed without shear reinforcement is only of the order 5 kg/cm². Hence design for shear.

Nominal reinforcement of transverse:

\[
\text{Ast(nominal)} = 0.24\% \; \text{Lt}
\]

Where,

\[
L = \text{length of the span}
\]

\[
t = \text{thickness of the shell}
\]

\[
= \frac{0.24}{100} \times 30 \times 10^3 \times 80
\]

\[
= 5760 \; \text{mm}^2
\]

Provide 20mm # bars.

Step: 7

Total bending moment for design:

BM due to eccentricity of TØ (comp. at base) = -64625.09 kgm

BM due to dead load and live load on shell = 118750.00 kgm

BM due to the self weight of the transverse = 27450.00 kgm

Total = 81574.91 kgm.

Total positive bending moment = 81574.91 kgm.

Step: 8

Direct tension and BM due to tension:

Tension = 36697.95 kg along the centre of the slab

Positive BM = 81574.91 kgm.
Step: 9

Design of reinforcement: (section subjected to tension and moment.)

The negative BM due to tension is less than the +ve moment, we can place the tension steel below the Neutral axis.

\[
\text{Ast for direct tension} = \frac{36697.95}{1400} = 26.21 \text{ cm}^2
\]

Use Fe250 grade steel, 16 mm # bar.

\[
\text{Nos. of bar} = \frac{2621}}{\pi(16)^2/4} \times 1000 = 13.035 \approx 14 \text{ nos.}
\]

\[
\text{Ast(pro)} = 14 \times \frac{\pi(16)^2}{4} = 2814.84 \text{ mm}^2
\]

Let us place these bars below N.A with an eccentricity of 9 cm B.M taken by these bar.

\[
= 28.14 \times 1400 \times 0.9 = 35456.4 \text{ kgm}
\]

Step: 10

Steel for balance moment

Balance moment = 81574.91 - 35456.4

Assume, \( jd = 0.86d \); \( d = 200 \text{ mm} \)

\[
\text{Ast} = \frac{46118.51 \times 100 \text{(kg/cm)}}{0.86 \times 200 \times 1400} = 19.15 \text{ cm}^2
\]
Nos. of bar = \( \frac{1915}{\pi (16)^{2}/4} \times 1000 \)

\[ = 9.52 \approx 10 \text{ nos.} \]

Provide 10 nos. of 16 mm # bars.

\[ \text{Ast(pro)} = 10 \times \frac{\pi (16)^{2}}{4} = 2010 \text{ mm}^2 \]

Step: 11

Check percentage of steel provide

Total steel = (14 + 10 =24 nos.) 16 mm.

\[ = (28.14 + 20.10) = 48.24 \text{ cm}^2 \]

\[ \% \text{ steel} = \frac{48.24 \times 100}{360 \times 30} = 0.447 \]

44\% > 34\%

hence safe.

Step: 12

Continuity bars between shell and supports.

In general, all the diagonal shear reinforcement placed in the shell is anchored into the transverse. In addition, we may provide 6 mm bar (as 20 cm × 20 cm fabric) to length of 75 cm (more than 2.5be = 2.5 × 30 = 70 cm) into the shell for anchorage of shell into the support.
DRAWINGS
Transverse

Cylindrical shell with edge beam
Interior view of shell and transverse lip

Over all view of the cylindrical shell
Conclusion:

Hence these shell structures are proved to be very safe with our design. This entire design was completed using approved code books IS 2210-1988, IS 456-2000, SP16 and ASCE Manual no.31. These structures from our design not only become economical but also they have huge advantages in many form to our living society. By this design we have studied many advantage and disadvantages of shell structures and also the specific needs of shell structures in some hurricane attacking areas, such places are to be noted here most of America mostly north America, Canada etc.. One of the cheapest demerits to be noted here, there can be no stories on the shell.
REFERENCE:
Design of Reinforced concrete shell and folded plates “P.C Varghese”
Design and construction of concrete shell roof “Ramaswamy”
Design graph for concrete shell roof “C.B White, C.B wilby”
Design RC elements “Krishna Raju”
Reinforced Concrete Halls “Hilal”